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# AN ADAPTIVE TECHNIQUE FOR ESTIMATING THE ATMOSPHERIC DENSITY PROFILE DURING THE A. E. MISSION

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ABSTRACT

This paper presents a technique for processing accelerometer data obtained during the A. E. missions in order to estimate the atmospheric density profile. A minimum variance, adaptive filter is utilized. The trajectory of the probe and probe parameters are in a consider mode where their estimates are unimproved but their associated uncertainties are permitted an impact on filter behavior. Simulations indicate that the technique is effective in estimating a density profile to within a few percentage points.

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# AN ADAPTIVE TECHNIQUE FOR ESTIMATING THE ATMOSPHERIC DENSITY PROFILE DURING THE A. E. MISSION

## INTRODUCTION

The AE-C, D, and E missions are designed to have nominal elliptical orbits with perigee altitudes as low as 120 km. At such altitudes the effect of aerodynamic heating and atmospheric drag are considerable. This together with the fact that the density profile in this portion of the atmosphere is not well known implies that data generated by instruments on the probe must be utilized to rapidly and accurately estimate atmospheric density as a function of altitude. Since telemetered accelerometer data will be available a certain well precededented procedure is suggested. The ideal gas law and the hydrostatic equation can be utilized to parameterize the density profile. The accelerometer data can then be processed in a differential correction program and the parameters of the atmospheric model can be estimated. The accuracy of the technique is limited by the fact that at any given time the altitude, velocity, mass, cross sectional area, and drag coefficient of the probe are all imperfectly known. Also the model used to parameterize the atmospheric density profile introduces further error since it is not a completely accurate reflection of reality. The best way to test the accuracy of the procedure is to construct a differential correction program for processing accelerometer data, generate simulated accelerometer data from an A. E. type trajectory using an atmospheric model different from what is assumed in the program, add the proper random component to the accelerometer and trajectory data, and determine how successful the procedure is in estimating the actual density profile.

This paper is a report on the results of such a test. The atmospheric density profile was parameterized by assuming that above a certain reference altitude  $H_0$ , temperature is linear with altitude. This assumption together with the ideal gas law and the hydrostatic equation permits one to obtain the density profile above  $H_0$  as a function of the density and temperature at  $H_0$  and the slope of the temperature profile above  $H_0$ . A recursive, minimum variance filter was designed to process accelerometer data and estimate these three parameters. The velocity and altitude of the probe at any given time along with mass, cross sectional area, and drag coefficient are treated in the filter as so called "consider" parameters. Thus the uncertainties of these parameters were permitted to have an impact on filter performance but their estimates were left unimproved. An adaptive feature was utilized to dynamically weight the accelerometer observations as a function of residuals. The addition of this feature was found to substantially improve the filter's performance.

A numerical integration scheme was used to generate A. E. type trajectories and the correct values of altitude, scalar velocity, and drag deceleration were recorded for various time points. With the aid of a random number generator, appropriate random components were added to these values as well as to probe mass, cross sectional area, and drag coefficient. The resultant simulated data was then processed by the filter and the estimated density profile was compared to the correct density profile. The process was repeated for two different atmospheres. The first atmosphere possessed a linear temperature profile above  $H_0$  as assumed by the filter atmospheric model. The results of this simulation demonstrate the optimum accuracy to be expected from the procedure. The second simulation was performed with an atmospheric model whose temperature profile above  $H_0$  was highly non linear. This simulation provides the results to be expected in the likely case that significant modeling errors are present.

Succeeding sections provide the mathematics of the atmospheric model and the recursive filter followed by details of the simulations.

## THE ATMOSPHERIC MODEL

Assume that above a reference altitude  $H_0$ , the Earth's atmosphere is spherically symmetric and perfectly mixed. Then the ideal gas law permits us to write

$$P(h) = \rho(h) R T(h)/m \quad (1)$$

where  $P(h)$ ,  $\rho(h)$ , and  $T(h)$  are respectively hydrostatic pressure, temperature, and density at altitude  $h$ . The symbols  $R$  and  $m$  respectively represent the ideal gas constant and the average molecular mass. The hydrostatic equation provides

$$\frac{dP(h)}{dh} = -g\rho(h) \quad (2)$$

where  $g$  is the gravitational constant (assumed independent of  $h$ ). From Equations 1 and 2, one can quickly derive

$$\frac{1}{\rho(h)} \frac{d\rho(h)}{dh} = -T(h) \left[ \frac{gm}{R} + \frac{dT(h)}{dh} \right] \quad (3)$$

Differential Equation 3 can be solved for  $\rho(h)$ ,  $h > H_0$ , only if  $\rho_0 = \rho(H_0)$  is known and if  $T(h)$ ,  $h > H_0$  is given. It is assumed in this paper that  $T(h)$  is linear above  $H_0$  with  $T_0 = T(H_0)$  and  $S$  the slope of  $T(h)$ . The parameters  $\rho_0$ ,  $T_0$  and  $S$  serve to define  $\rho(h)$ ,  $h > H_0$ . To see how, integrate Equation 3 from

$H_0$  to some higher altitude  $h$ . This immediately yields

$$\text{Log}_e \left[ \frac{\rho(h)}{\rho_0} \right] = -\text{Log}_e \left[ \frac{T(h)}{T_0} \right] - \frac{gm}{R} \int_{H_0}^h \frac{dh}{T(h)} \quad (4)$$

and by taking advantage of the linear form of  $T(h)$  one can easily derive from Equation 4,

$$\rho(h) = \rho_0 \left[ \frac{T_0}{T_0 + S(h - H_0)} \right]^{1+gm/RS}, h > H_0 \quad (5)$$

### PROCESSING ACCELEROMETER DATA

The scalar acceleration due to drag experienced by a probe in an atmosphere is given by

$$a = C V^2 \rho(h) \quad (6)$$

where  $\rho(h)$  is the atmospheric density at altitude  $h$ ,  $V$  is the scalar velocity of the probe and  $C$  is given by

$$C = \frac{1}{2} A C_D / n \quad (7)$$

where  $A$  is the cross sectional area of the probe,  $C_D$  is the drag coefficient and  $n$  is the probe mass. In the case of an A. E. probe, the drag coefficient is determined analytically as a function of angle of attack by assuming the probe to be a cylinder. The cross sectional area and mass are continuously monitored. A tri-axial set of accelerometers are assumed to be installed on the probe. These accelerometers measure three mutually perpendicular components of the acceleration due to drag. The root mean square value of the three accelerometers provides an observation of the scalar acceleration due to drag as symbolized on the left side of Equation 6. Combining Equations 5 and 6 yields

$$a = C V^2 \rho_0 \left[ \frac{T_0}{T_0 + S(h - H_0)} \right]^{1+gm/RS} \quad (8)$$

The altitude  $h$  and scalar velocity  $V$  of the probe at the instant an accelerometer reading is obtained is assumed available as the output of an orbit determination program. The problem can now be stated. Given a series of measurements of the left side of Equation 8, determine the most efficient filter for processing

these observations to obtain estimates of  $\rho_0$ ,  $T_0$ , and  $S$ . The development of such a filter must take into account uncertainties in  $C$ ,  $h$ , and  $V$  as well as uncertainties in the observations. For reasons of computational convenience it is desired that the resultant filter be both linear and recursive. In order to utilize results of linear filtering theory it is necessary to assume a linear approximation to Equation 8. Let  $T_i$  be the time at which the  $i^{\text{th}}$  accelerometer measurement is taken. Then define the following symbols

$$X = \begin{bmatrix} \rho_0 \\ T_0 \\ S \end{bmatrix}, \quad Y_i = \begin{bmatrix} h_i \\ V_i \\ C_i \end{bmatrix} \quad (9)$$

where  $h_i$ ,  $V_i$ , and  $C_i$  are respectively the values of  $h$ ,  $V$ , and  $C$  of Equation 8 at time  $T_i$ . Equation 8 can be symbolized as

$$a_i = f(X, Y_i) \quad (10)$$

where  $a_i$  is the true acceleration due to drag at time  $T_i$ . Let  $X_n$  and  $Y_{i,n}$  be nominal estimates of  $X$  and  $Y_i$  which may be thought of as the best estimates of  $X$  and  $Y_i$  immediately before the  $i^{\text{th}}$  accelerometer measurement is processed. A linear approximation to Equation 10 must take the form

$$a_i = G_1 X + G_2 Y_i + G_3 \quad (11)$$

For our purposes it is not necessary to know a precise value for  $G_3$ . The estimates used for matrices  $G_1$  and  $G_2$  are

$$G_1 = \left. \frac{\partial f(X, Y_i)}{\partial X} \right|_{X=X_n, Y_i=Y_{i,n}}, \quad G_2 = \left. \frac{\partial f(X, Y_i)}{\partial Y_i} \right|_{X=X_n, Y_i=Y_{i,n}} \quad (12)$$

At time  $T_i$ , an estimate  $\hat{Y}_i$  of  $Y_i$  is available with statistics.

$$\hat{Y}_i = Y_i + \gamma_i, E(\gamma_i) = 0, E(\gamma_i \gamma_i^T) = \rho_i, E(\gamma_i \gamma_j^T) = 0, i \neq j \quad (13)$$

Since the filter is assumed recursive, an estimate  $\hat{X}_{i-1}$  of  $X$  based on observations obtained previous to time  $T_i$  is available with statistics

$$\hat{X}_{i-1} = X + \alpha_{i-1}, E(\alpha_{i-1}) = 0, E(\alpha_{i-1} \alpha_{i-1}^T) = P_{i-1} \quad (14)$$

The matrices  $\rho_i$  and  $P_{i-1}$  are presumed known. At time  $T_i$  a direct observation  $\hat{a}_i$  of  $a_i$  is obtained with statistics

$$\begin{aligned} \hat{a}_i &= a_i + \nu_i + \tau_i, E(\nu_i) = E(\tau_i) = 0, E(\nu_i \tau_i) = E(\nu_i \nu_j) = E(\tau_i \tau_j) = 0, i \neq j \\ E(\nu_i^2) &= Q, E(\tau_i^2) = R_i \end{aligned} \quad (15)$$



The random variable  $\nu_i$  represents the inherent instrument noise and its variance  $Q$  is considered as known. The random variable  $\tau_i$  represents observation noise due to modeling errors such as inaccuracies in Equation 8 and errors caused by the linearization symbolized by Equation 11. Its variance  $R_i$  is not known a priori but is determined by the residuals of the filter in a way which will be discussed later.

Since the filter is constrained to be linear, the estimate  $\hat{X}_i$  is obtained by processing observation  $\hat{a}_i$  must take the form

$$\hat{X}_i = H_1 \hat{a}_i + H_2 \hat{X}_{i-1} + H_3 \hat{Y}_i + H_4 \quad (16)$$

where  $H_1$ ,  $H_2$ ,  $H_3$ , and  $H_4$  are matrices of the proper dimension. At this point the unbiasedness condition is imposed on the filter. In short, we insist that

$$E(\hat{X}_i) = X \quad (17)$$

Condition 17 has important consequences. By using Equations 11 through 15, Equation 16 can be written as

$$\hat{X}_i = [H_1 G_1 + H_2] X + [H_1 G_2 + H_3] \hat{Y}_i + H_1 G_3 + H_4 + H_1 \nu_i + H_2 \alpha_{i-1} + H_3 \gamma_i \quad (18)$$

The only way to insure that Equation 17 is satisfied is to impose conditions

$$H_1 G_1 + H_2 = I, \quad H_1 G_2 + H_3 = 0, \quad H_1 G_3 + H_4 = 0 \quad (19)$$

Equations 19 permits us to represent matrices  $H_2$ ,  $H_3$ , and  $H_4$  in terms of  $H_1$ . Equation 16 can then be written as

$$\hat{X}_i = \hat{X}_{i-1} + H_1 [\hat{a}_i - G_1 \hat{X}_{i-1} - G_2 \hat{Y}_i - G_3] \quad (20)$$

The expression  $G_1 \hat{X}_{i-1} + G_2 \hat{Y}_i + G_3$  may be viewed as the best estimate of  $a_i$  previous to processing  $\hat{a}_i$  and assuming linear approximation 11 to Equation 10. Thus Equation 20 represents the filter estimate  $\hat{X}_i$  as the sum of the previous estimate  $\hat{X}_{i-1}$  and the product of the vector  $H_1$  by the difference between the observed value  $\hat{a}_i$  of  $a_i$  and the computed best estimate value of  $a_i$  based on estimates  $\hat{X}_{i-1}$  and  $\hat{Y}_i$  of  $X$  and  $Y_i$ . This suggests that we define

$$a_i^c = f(\hat{X}_{i-1}, \hat{Y}_i) \quad (21)$$

where the function "f" is defined by Equation 10 and is explicitly represented by the right side of Equation 8. Equation 20 can be restated as

$$\hat{X}_i = \hat{X}_{i-1} + H_1 (\hat{a}_i - a_i^c) \quad (22)$$

where  $H_1$  is an as yet undetermined vector. To determine  $H_1$  we impose the so called minimum variance condition on  $\hat{X}_i$ . Define

$$P_i = E \left( (\hat{X}_i - X) (\hat{X}_i - X)^T \right) \quad (23)$$

Since the expected value of  $\hat{X}_i$  is  $X$ ,  $P_i$  represents the covariance matrix of  $\hat{X}_i$ . We intend to choose a value for  $H_1$  which minimizes the trace of  $P_i$ . The result, ignoring the non linearity of Equation 21, will be a linear, unbiased, minimum variance filter. Equations 18 and 19 permit one to write

$$\hat{X}_i = X + H_1 \nu_i + (I - H_1 G_1) \alpha_{i-1} - H_1 G_2 \gamma_i + H_1 \tau_i \quad (24)$$

Hence

$$P_i = H_1 Q H_1^T + H_1 R_i H_1^T + (I - H_1 G_1) P_{i-1} (I - H_1 G_1)^T + H_1 G_2 \rho_i G_2^T H_1^T \quad (25)$$

Define

$$\begin{aligned} \delta_1 &= Q + R_i + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \\ \delta_2 &= P_{i-1} G_1^T \end{aligned} \quad (26)$$

Then Equation 25 can be written as

$$P_i = H_1 \delta_1 H_1^T - H_1 \delta_2^T H_1^T + P_{i-1} \quad (27)$$

The problem now is to obtain the value of  $H_1$  which minimizes the trace of  $P_i$ . Assume the following change of variables

$$H_1 = \delta_2 \delta_1^{-1} + \Delta \quad (28)$$

Then Equation 27 can be written

$$P_i = P_{i-1} - \delta_2 \delta_1^{-1} \delta_2^T + \Delta \delta_1 \Delta^T \quad (29)$$

Since  $\delta_1$  is necessarily a positive number, the trace of  $P_i$  is minimized when  $\Delta = 0$  and the value of  $H_1$  which provides a minimum variance filter is

$$H_1 = \delta_2 \delta_1^{-1} \quad (30)$$

And from Equations 29 and 30, the covariance matrix of the minimum variance filter is

$$P_i = P_{i-1} - H_i \delta_2^T \quad (31)$$

The linear, unbiased, minimum variance, recursive filter can be given as

$$a_i^c = f(\hat{X}_{i-1}, \hat{Y}_i) \quad (32a)$$

$$\delta_1 = Q + R_i + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \quad (32b)$$

$$\delta_2 = P_{i-1} G_1^T \quad (32c)$$

$$H = \delta_2 \delta_1^{-1} \quad (32d)$$

$$\hat{X}_i = \hat{X}_{i-1} + H(\hat{a}_i - a_i^c) \quad (32e)$$

$$P_i = P_{i-1} - H \delta_2^T \quad (32f)$$

Equations 32 are not complete since the value of  $R_i$  is undetermined. A mode of estimating  $R_i$  as a function residuals will now be developed. The procedure will again be a recursive one. Hence we assume that an estimate  $\hat{R}_{i-1}$  of  $R_{i-1}$  was obtained at time  $T_{i-1}$  with statistics

$$\hat{R}_{i-1} = R_{i-1} + \kappa_{i-1}, E(\kappa_{i-1}) = 0, E(\kappa_{i-1}^2) = \beta_{i-1} \quad (33)$$

Assume further that the relationship between  $R_i$  and  $R_{i-1}$  is

$$R_i = R_{i-1} + \pi_i, E(\pi_i) = 0, E(\pi_i^2) = \omega \quad (34)$$

Define the residual of the filter at time  $T_i$  as

$$\epsilon_i = \hat{a}_i - a_i^c \quad (35)$$

Using the linear approximation implied by Equation 11, Equation 35 can be written

$$\epsilon_i = \nu_i + \tau_i - G_1 \alpha_{i-1} - G_2 \gamma_i \quad (36)$$

Next define  $\lambda_i$  as the variance of  $\epsilon_i$

$$\lambda_i = E(\epsilon_i^2) \quad (37)$$

Equations 36 and 37 lead to

$$\lambda_i = Q + R_i + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \quad (38)$$

Equations 34 and 38 can be viewed respectively as linear state and observation equations. If an unbiased observation of  $\lambda_i$  could be obtained it could then be processed in a standard minimum variance fashion and a linear, unbiased, minimum variance, recursive filter could be obtained for estimating  $R_i$ . It can be shown that the square of the residual  $\epsilon_i$  obtained at time  $T_i$  is a chi-square random variable with expectation equal to  $\lambda_i$  and variance equal to  $2\lambda_i^2$ . See [1] for details. The number  $\epsilon_i^2$  can now be treated as an observation of  $\lambda_i$ . Its variance can be estimated from Equation 38 by substituting  $\hat{R}_{i-1}$  for  $R_i$  on the right side. The result is

$$E[(\epsilon_i^2)(\epsilon_i^2)] \approx 2 [Q + \hat{R}_{i-1} + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T]^2 \quad (39)$$

The resultant estimator for  $R_i$  is given by the following recursion relations

$$\chi = Q + \hat{R}_{i-1} + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \quad (40a)$$

$$q = 2 \chi^2 \quad (40b)$$

$$z = \beta_{i-1} + \omega \quad (40c)$$

$$\mathcal{H} = z(z + q)^{-1} \quad (40d)$$

$$\hat{R}_i = \hat{R}_{i-1} + \mathcal{H}(\epsilon_i^2 - \chi) \quad (40e)$$

$$\beta_i = z(1 - \mathcal{H})$$

Again the details can be found in [1]. It is instructive to see how Equations 40 interface with Equations 32 to form the complete filter. Given the mathematical model of Equations 11 through 15 and given  $\hat{X}_{i-1}$ ,  $\hat{Y}_{i-1}$ ,  $P_{i-1}$ ,  $\hat{R}_{i-1}$ ,  $\beta_{i-1}$ , the minimum variance recursive filter is

$$a_i^c = f(\hat{X}_{i-1}, \hat{Y}_i) \quad (41a)$$

$$\epsilon_i = \hat{a}_i - a_i^c \quad (41b)$$

$$\chi = Q + \hat{R}_{i-1} + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \quad (41c)$$

$$q = 2 \chi^2$$

$$z = \beta_{i-1} + \omega \quad (41e)$$

$$\mathcal{H} = z(z + q)^{-1} \quad (41f)$$

$$\hat{R}_i = \hat{R}_{i-1} + \mathcal{H}(\epsilon_i^2 - \chi) \quad (41g)$$

$$\beta_i = z(1 - \mathcal{H}) \quad (41h)$$

$$\delta_1 = Q + \hat{R}_i + G_1 P_{i-1} G_1^T + G_2 \rho_i G_2^T \quad (41i)$$

$$\delta_2 = P_{i-1} G_1^T \quad (41j)$$

$$H = \delta_2 \delta_1^{-1} \quad (41k)$$

$$\hat{X}_i = \hat{X}_{i-1} + H(\hat{a}_i - a_i^c) \quad (41l)$$

$$P_i = P_{i-1} - H \delta_2^T \quad (41m)$$

The structure of this set of recursion relations may become clearer if one views Equations 41 as a pair of interlocking minimum variance filters. The inner filter defined by Equations c through h estimates observation noise variance  $R_i$  and the outer filter defined by Equations a through b and i through m estimates the state  $X$ . The outer filter passes the residual  $\epsilon_i$  to the inner filter which processes  $\epsilon_i^2$  as an observation and estimates  $R_i$ . The number  $\hat{R}_i$  is then treated by the outer filter as observation noise variance and an estimate  $\hat{X}_i$  of  $X$  is obtained.

The filter is adaptive in the sense that a few large residuals will cause the inner filter to return a large estimate of observation noise variance with a resultant downgrading of the impact of observations on the estimate. In this sense the filter adapts its behavior to the presence of modeling errors.

An adaptive feature when used in this way amounts to a dynamic weighting scheme which automatically downgrades the impact of data points which are associated with large residuals. Thus the necessity of editing the data before processing is eliminated. This is important with reference to real time applications. Notice also that the filter defined by Equations 41 does not utilize the data to produce an improvement in the estimate  $\hat{Y}_i$  of  $Y_i$ . This means that the accelerometer data is used to improve the estimate of the atmosphere rather than the actual trajectory or the parameters associated with the probe. The uncertainties of the trajectory and the probe parameters as given by covariance matrix  $\rho_i$  do, however,

influence the estimate  $\hat{X}_i$  of  $X$ . The parameters which define the vector  $Y_i$  are said to be in a "consider mode" rather than a "solved for" mode.

## SIMULATIONS

Although the results of linear filtering theory were used to generate the results of the previous section, the filter defined by recursion relations 41 is non linear. Consequently it is not possible to evaluate its performance by ensemble studies. If one is to know how such a filter will perform during the actual mission, simulations must be performed. For the first simulation to be discussed a numerical integrator was employed to generate an A. E. type trajectory. The atmospheric model was obtained by assuming the ideal gas law and the hydrostatic equation and a temperature profile which is a polygonal arc with four breakpoints the highest of which at the reference altitude. Since for this model the temperature above the reference altitude is linear with altitude, no modeling errors were present for this simulation. At discrete time points, so called true values of altitude, velocity, and acceleration due to drag were recorded. A random number generator was utilized to add appropriate random components to these numbers. Random components were also added to the values of the cross sectional area, mass, and drag coefficient that were used to generate the true data. Recursion relations 41 were translated into a computer program which processed the simulated data to yield an estimate of the atmospheric density profile. The performance of the filter was measured by the percentage error between the estimated density and the actual density as a function of altitude.

The perigee altitude for the trajectory was 135 km. From 7 minutes before perigee until 7 minutes after perigee, a total of 175 accelerometer measurements were obtained. The uncertainty figures used in the simulations are provided in Table 1. The reference altitude was chosen at 100 km. For this estimation technique it is necessary to begin with an a priori estimate of the atmospheric density profile. For this simulation a very poor a priori estimate was used. This was done for two reasons. First, the filter's performance would not be very impressive if in order to obtain a good estimate of the density profile it must begin with a good estimate. The second reason is that in pseudo-real time applications such as this the question of the stability of the filter becomes important. If a poor a priori estimate is provided to a recursive filter a frequent result is filter divergence and a new a priori estimate must be tried. This involves a searching procedure which can become lengthy.

The results of the simulation are displayed in Figure 1 which is a plot of the estimated density minus the true density divided by the true density and multiplied

Table 1

## Standard Deviations Used in Simulation

Parameter	Standard Deviation
Altitude	500 m
Velocity	4 m/sec
Accelerometer Reading	$5 (10)^{-5} \text{ m/sec}^2$
Probe Cross Sectional Area	$0.16 \text{ m}^2$
Probe Mass	6.5 kg
Probe Drag Coefficient	0.13

by 100. It is seen that between 130 km and 200 km the filter succeeds in estimating the density with an average error of less than 2%. In the same region the a priori estimate of the density profile would have introduced an average error of well over 600%.

In the simulation which produced Figure 1 the assumption that above the reference altitude temperature is linear is satisfied. During the actual mission this is not likely to be the case. In fact present information [2] indicates that temperature above 100 km is likely to possess a negative second derivative with regard to altitude. This difficulty could be solved by modeling several breakpoints above 100 km and solving for the extra slopes. But this leads to a more complicated filter. It is interesting to observe how the present filter performs in estimating the density profile of an atmosphere whose temperature profile is not linear above the reference altitude. The second simulation discussed in this paper was performed identically to the first with the exception that the atmospheric model had six temperature breakpoints the last two of which were at 125 km and 160 km. The slopes were chosen so that the resulting temperature profile resembled the curves shown in [2]. The results are shown in Figure 2 which can be interpreted in the same way as Figure 1. The average error between 130 km and 200 km has increased to about 3%. This suggests that an atmospheric model more elaborate than what is used in the present filter may not be necessary in order to adequately estimate the density profile. The same simulations were completed without the adaptive feature. The average error between 130 km and 200 km for simulation one was about 3% and for simulation

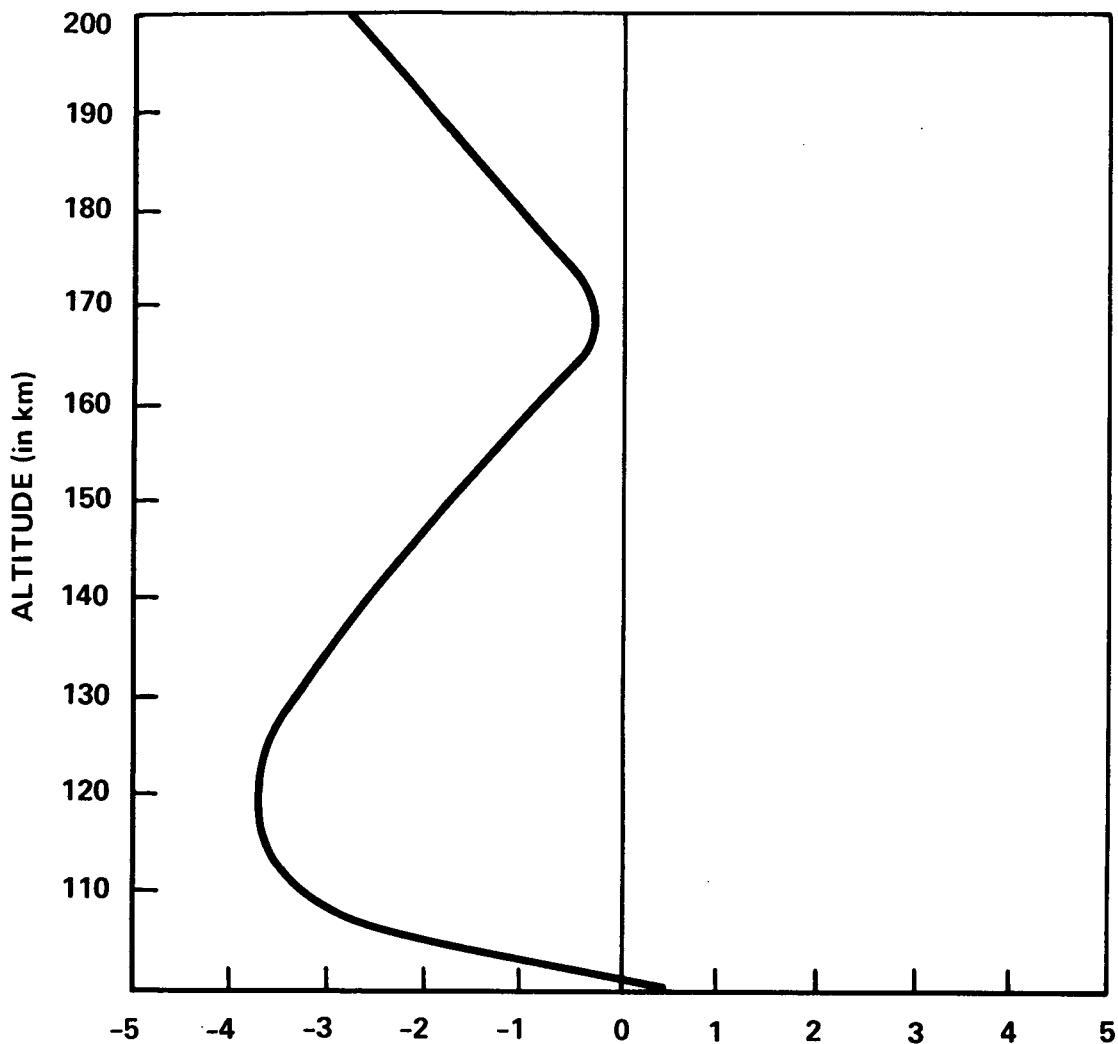


Figure 1. Percentage Error of Density Estimate versus Altitude for First Simulation

2 about 4.5%. The adaptive feature appears to be well worth the added complexity it introduces. This added complexity is not great as seen by the fact that with the adaptive feature included the filter is capable of processing over 1,500 accelerometer readings per minute on a modern high speed computer.

## CONCLUSIONS

Accelerometer data collected during the A. E. mission along with trajectory data can be processed in order to yield an estimate of the atmospheric density



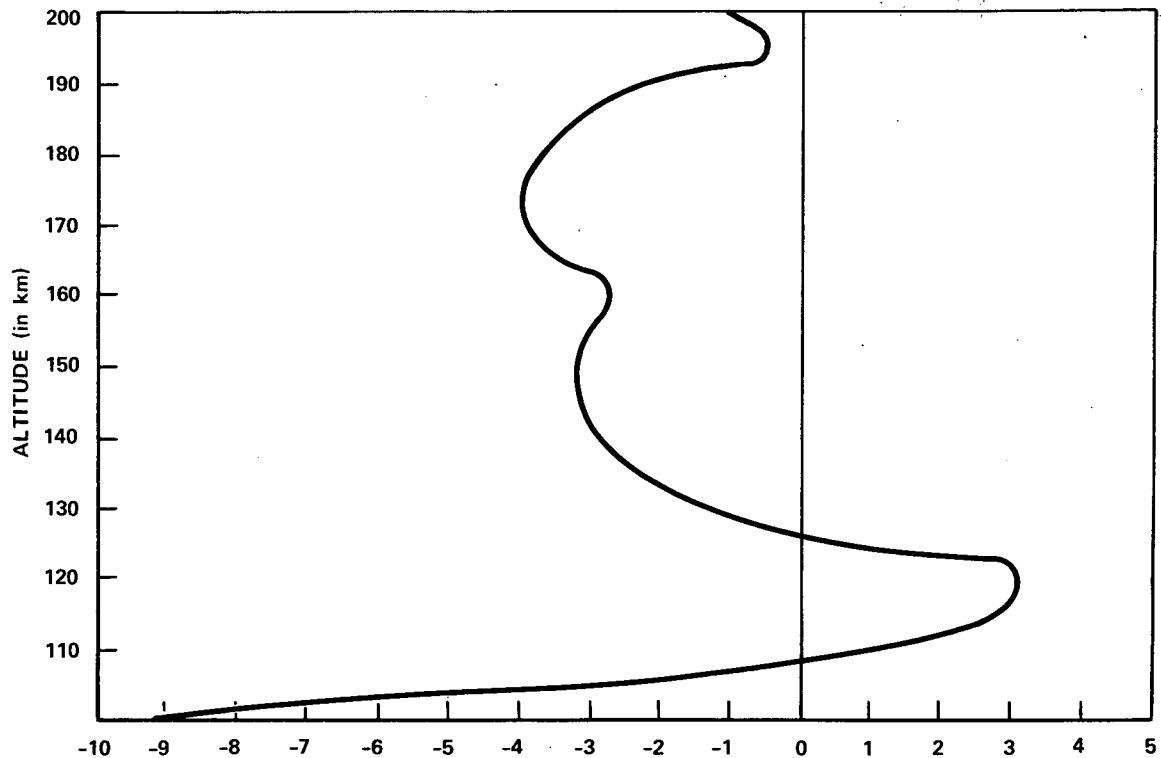


Figure 2. Percentage Error of Density Estimate versus Altitude for Second Simulation

profile that should be accurate to within a few percentage points. In this paper the filter used to process accelerometer data was minimum variance, recursive, and with an adaptive observation noise estimator. The altitude, velocity, and probe parameters were in a consider mode where their estimates were unimproved but their associated uncertainties were permitted to have an impact on the filter estimate. The atmospheric model used by the filter characterize the density profile with just three parameters. This appears to be adequate since simulations show that even when the modeling assumptions are seriously violated the filter performs well. Simulations also show that the filter converges to a good estimate even when the a priori estimate of the density profile is in error by several hundred percentage points. The adaptive estimation of observation was found to add substantially to the accuracy of the technique.

As designed the filter should be relatively insensitive to data dropouts and to badly biased data points. Its speed should be adequate for real time applications and its accuracy may be adequate for post flight analysis.

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